GRAIL orbit determination in the Bernese GNSS Software

first results with combined Doppler and inter-satellite KBRR

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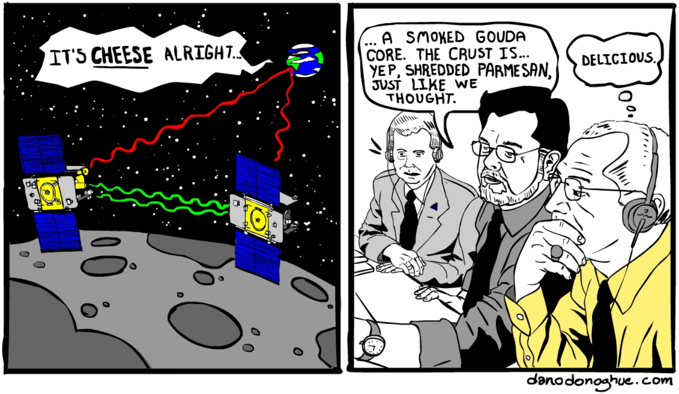
Outline

The GRAIL mission

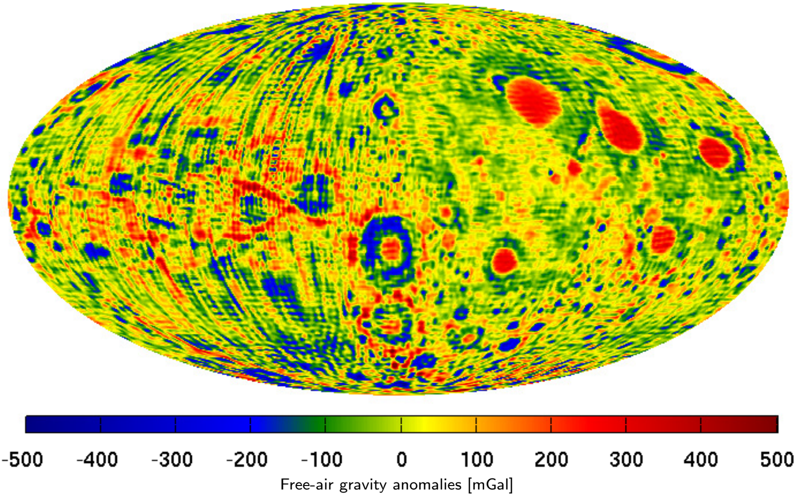
Science objectives



- Determine structure of lunar interior, from crust to core
  - → Subsurface structure of impact basins, mascons, ...
- Understand (asymmetric) thermal evolution of Moon

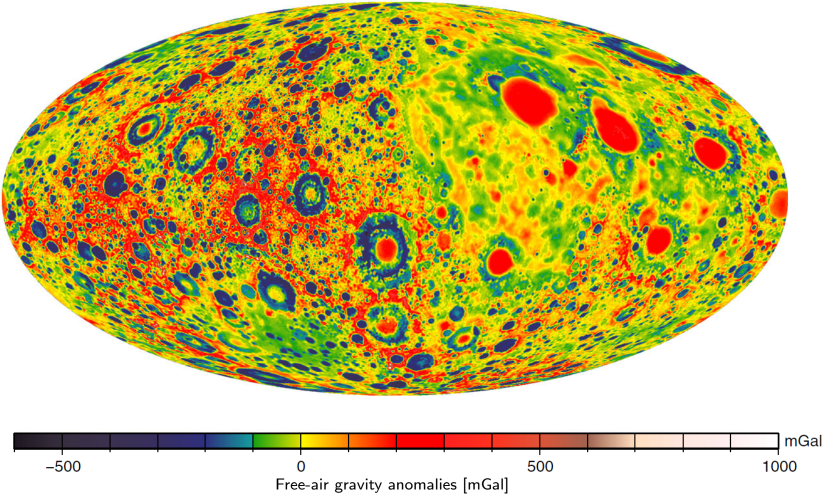
The GRAIL mission

Pre-GRAIL lunar gravity missions: Lunar Prospector (NASA, 1998-99) JGL165P1, SELENE (JAXA, 2007-09) SGM150J

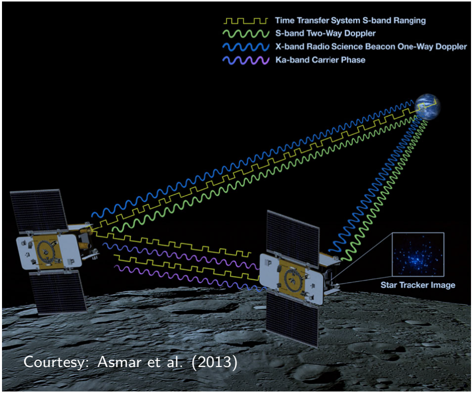


The GRAIL mission

GRAIL: Latest official $l_{\text{max}} = 900$ gravity field models:
GRGM900C (Lemoine et al., 2014), GL0900C (Konopliv et al., 2014)



The GRAIL mission: Satellite signals



- S-band ($\sim [2] GHz$) for 2-way Doppler tracking by NASA Deep Space Network (DSN)
- X-band ($\sim [8] GHz$) for 1-way Doppler tracking
- Ka-band ($\sim [32] GHz$) inter-satellite link

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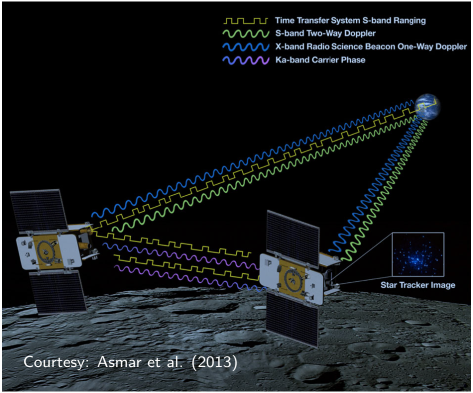

Courtesy: Asmar et al. (2013)

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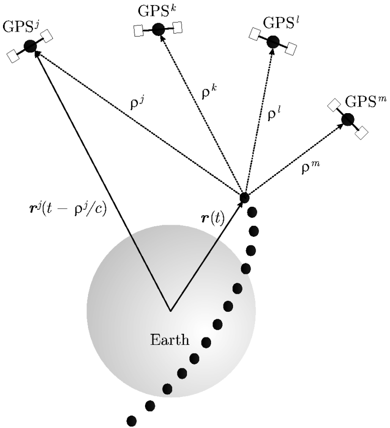
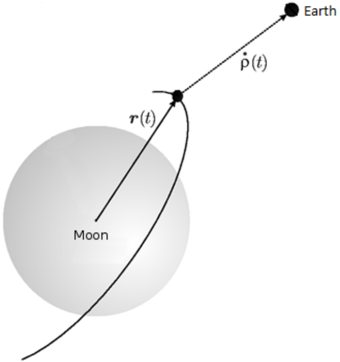
Our motivation: Why not adapt our procedures for the processing of GRACE data (Ka-band, etc.) to GRAIL, get experienced in this new environment and eventually provide an independent lunar gravity field solution?

GRACE vs. GRAIL





GRACE: Kinematic positions using GPS observations

GRAIL: DSN Doppler tracking (near-side only) yields positions

The GRAIL mission: Available data

Selection of available data for our activities:

- 1-way (X-band) and 2-way (S-band) Doppler data

Current Bernese Software: GNSS (LEO) and SLR for orbit and gravity field determination → development of DSN Doppler capability !

- Ka-band range data: Ka-band range rate (KBRR)
  - → [5]s-sampling in primary, [2]s-sampling in extended mission phase

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- Ka-band range data: Ka-band range rate (KBRR)
  → [5]s-sampling in primary, [2]s-sampling in extended mission phase
- Reduced-dynamic positions (GNI1B) of GRAIL-A and GRAIL-B (by-product of JPL gravity field estimation)
  → [5]s-sampling in primary and extended mission phase

Use of the GNI1B positions as pseudo-observations : **back to GRACE-like scenario** → generalization of the force model

The Celestial Mechanics Approach

(implemented in the Bernese GNSS Software)

The Celestial Mechanics Approach (CMA)

Selenocentric equation of motion for satellite i

$$\ddot{\mathbf{r}}_i = -GM_M \frac{\mathbf{r}}{r^3} + \mathbf{f}(t, \mathbf{r}, \dot{\mathbf{r}}, q_1, ..., q_d)$$

$$\mathbf{f} = \nabla V + \mathbf{a}_b + \mathbf{a}_t + \mathbf{a}_r + \mathbf{a}_e + \mathbf{a}_n$$

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V Lunar gravity potential:

$$V(r, \lambda, \phi) = \frac{GM_M}{r} \sum_{l=1}^{l_{\mathrm{max}}} \left(\frac{R_M}{r}\right)^l \sum_{m=0}^{l} \bar{P}_{lm}(\sin\phi) \left(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right)$$

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$\mathbf{a}_b$ 3rd body perturbations (Earth, Sun, Jupiter, Venus, Mars, according to JPL ephemerides DE421)

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$\mathbf{a}_t$ Tidal deformation of Moon due to Earth and Sun. IERS2010 conventions:

$$\Delta\bar{C}_{lm} - i\Delta\bar{S}_{lm} = \frac{k_{lm}}{2l+1} \sum_{j=2}^{3} \frac{GM_j}{GM_M} \left(\frac{R_M}{r_j}\right)^{l+1} \bar{P}_{lm}(\sin\Phi_j) e^{-im\lambda_j}$$

Use Love numbers k_{20}, k_{21}, k_{22} and k_{30} from Lemoine et al. (2013), neglect change of deg. 4 coefficients due to deg. 2 tides.

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$\mathbf{a}_r$ Relativistic corrections

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- $\mathbf{a}_r$ Relativistic corrections
- $\mathbf{a}_e$ Empirical forces
- $\mathbf{a}_n$ Non-gravitational accelerations, especially solar radiation pressure $\rightarrow$ **not yet explicitly modeled**

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Orbit parametrization: $\mathbf{r}_i(t; a, e, i, \Omega, \omega, u_0; Q_1, ..., Q_d, P_1, ..., P_s)$

Q_i: Dynamic parameters (general and arc-specific)
P_i: Pseudo-stochastic parameters (pulses, all directions)

The Celestial Mechanics Approach (CMA)

- Numerical integration (with a priori parameters) of equations of motion and variational equations.
- Set up of normal equations (NEQs) **for each observation type** (Doppler, KBRR, positions, ...) on a daily basis.
- Combination of the NEQs with appropriate weighting. $\rightarrow$ **w.r.t. relative accuracy of observations**
- NEQ manipulation (e.g., preelimination of parameters and accumulation to weekly, monthly, etc... NEQs)
- NEQ inversion $\rightarrow$ simultaneous solution for the improved parameters (orbit, gravity field coefficients, etc...)

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[Beutler, G. et al., *The celestial mechanics approach: theoretical foundations.* Journal of Geodesy, 2010]

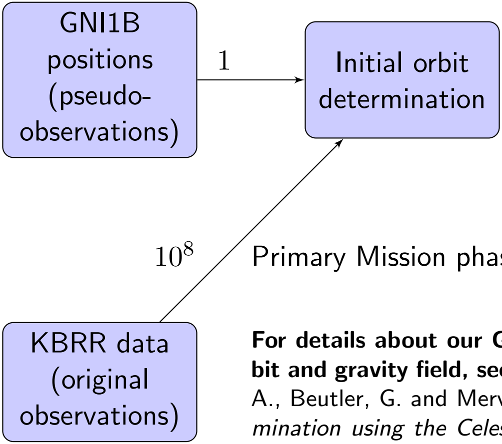
[Jäggi, A. et al., *Pseudo-Stochastic Orbit Modeling Techniques for Low-Earth Orbiters.* Journal of Geodesy, 2006.]

Setup and results using Doppler+KBRR data

(New implementation in Bernese GNSS Software, preliminary study)

Doppler orbit determination

Use the GNI1B positions as pseudo-observations for an initial orbit determination for GRAIL-A and GRAIL-B.
Add the Ka-band range rate data to improve orbit determination.



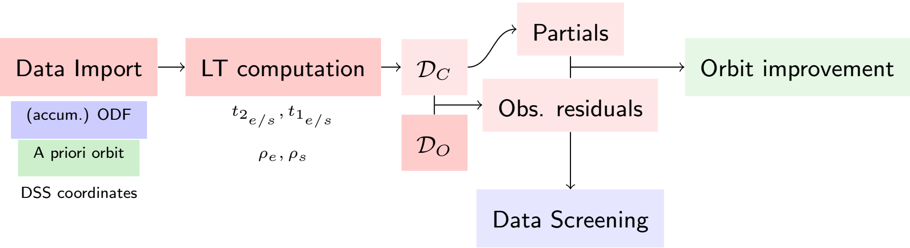
Primary Mission phase (March 2012 - May 2012)

For details about our GNI1B + KBRR solution for orbit and gravity field, see: [Arnold, D., Bertone, S., Jäggi, A., Beutler, G. and Mervart, L. *GRAIL gravity field determination using the Celestial Mechanics Approach*, Icarus, 2015]

DSN Doppler data processing

Doppler model based on [Moyer, 2000]. It includes :

- Tracking stations Earth-fixed coordinates (Folkner W., 1997+)
- Earth rotation (IERS2010)
- planetary ephemeris (DE421, . . .)
- Space-time frame transformations (IAU2010)
- Relativistic effects (Shapiro, . . .)
- Atmospheric delay (troposphere, ionosphere)



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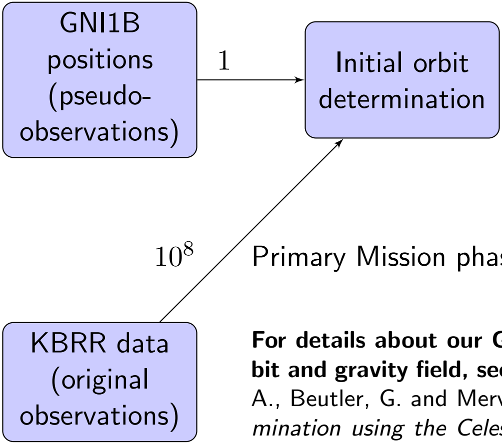

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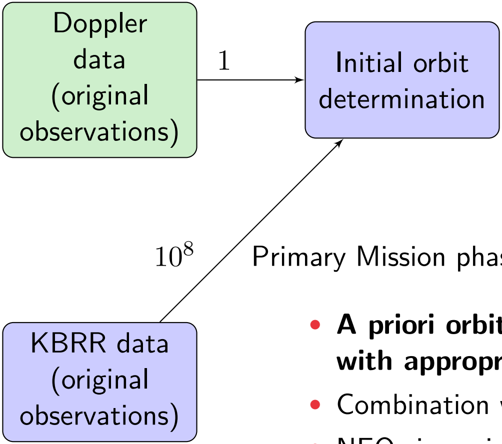
Doppler orbit determination

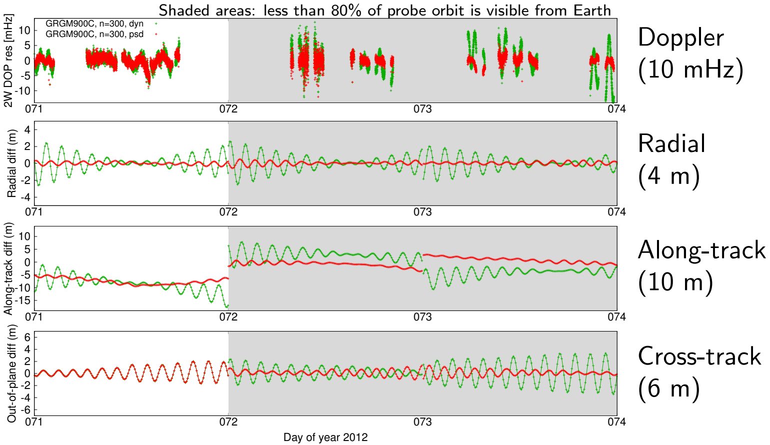
Goal: Replace GNI1B positions by original DSN Doppler observations.



Primary Mission phase (March 2012 - May 2012)

- **A priori orbit: fit of Doppler observations with appropriate parametrization**
- Combination with KBRR daily NEQs
- NEQs inversion and solution

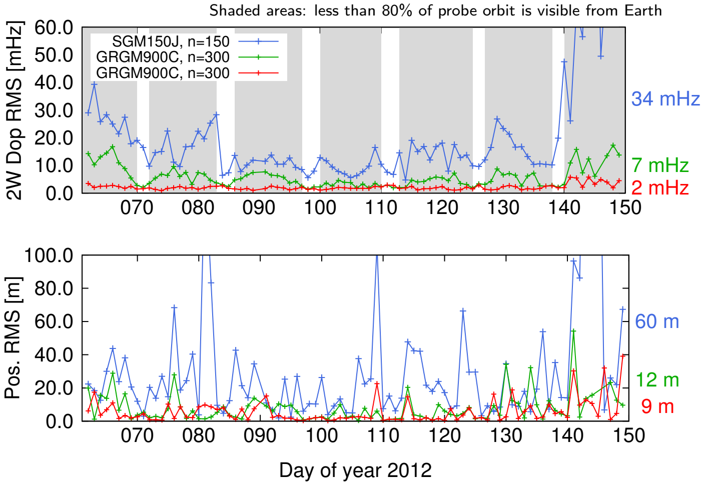
Two-way S-band Doppler



We consider 3 background models over the primary mission (PM):

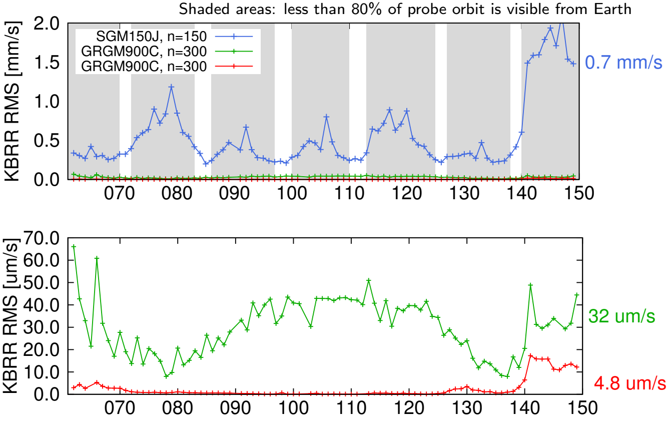
- GRGM900C (up to d/o 300), dynamic modeling
- GRGM900C (up to d/o 300), acc: const A + opr R , pulses: 30' AO
- SGM150J (SELENE mission), dynamic modeling

Doppler only solution



Daily RMS values of Doppler residuals and orbit differences of GRAIL-A (PM) w.r.t. GNI1B: **results comparable with GEODYN and GINS with similar setup** (*private communications: S. Goossens and J.-C. Marty*).

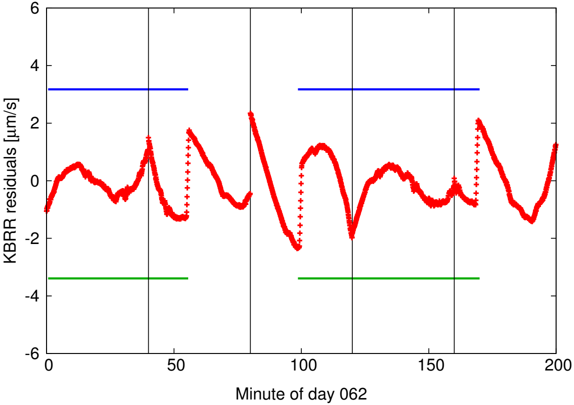
Combined orbit: KBRR residuals



Daily RMS values of KBRR residuals for GRAIL-A (PM), using different gravity field models and parametrizations. (**nominal:** $0.1\mu m/s$, see Kruizinga G., AAS 2013)

- Days $140 - 150$ at lower altitude $\rightarrow$ larger residuals

Combined orbit: KBRR residuals

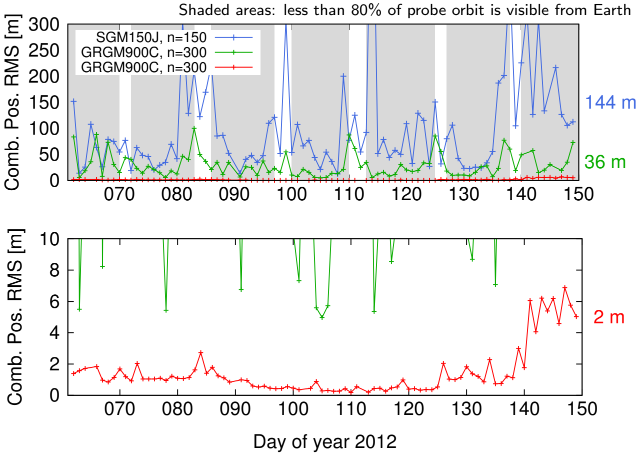


Visible effects:

- impact of the pseudo-stochstic pulses (here, every 40');
- solar radiation pressure at light/shadow transitions.

Improve force modelling (SRP) to further reduce residuals!

Combined orbit: position differences



Daily RMS values of orbit differences of GRAIL-A (PM) w.r.t. GNI1B, using several gravity fields and parametrizations. (see Kruizinga G., AAS 2013)

- Days $140 - 150$ at lower altitude $\rightarrow$ larger residuals

Gravity field determination

Goal: Gravity field determination from initial orbit determination

GNI1B / Doppler

Initial orbit determination

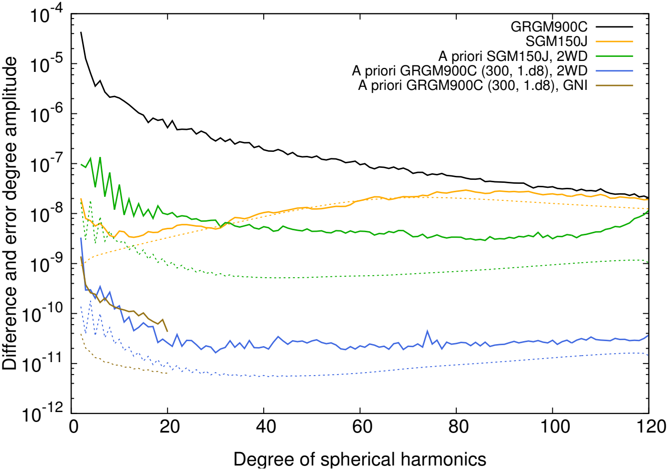
Orbit and gravity field determination

KBRR data (original observations)

- NEQs stacking up to the whole mission (pre-elimination of orbit parameters)
- NEQ solution for gravity field coefficients C_{lm} and S_{lm}
- d/o 200 solution from GNI1B + KBRR
- d/o 120 solution from 2-way Doppler + KBRR

Gravity field determination: 2WDOP, $l_{\mathbf{m}} = 120$

Difference degree amplitudes : $\Delta_l = \sqrt{\frac{1}{2l+1} \sum_{m=0}^{l} \left(\Delta\bar{C}_{lm}^2 + \Delta\bar{S}_{lm}^2\right)}$



a priori SGM150J: bad low degrees, more iterations needed

a priori GRGM900C (d/o 300): same level as GNI+KBRR solution

- First d/o 120 solution from original observations
- Need to improve solutions starting from "poor" a priori gravity fields.

Conclusion & Outlook

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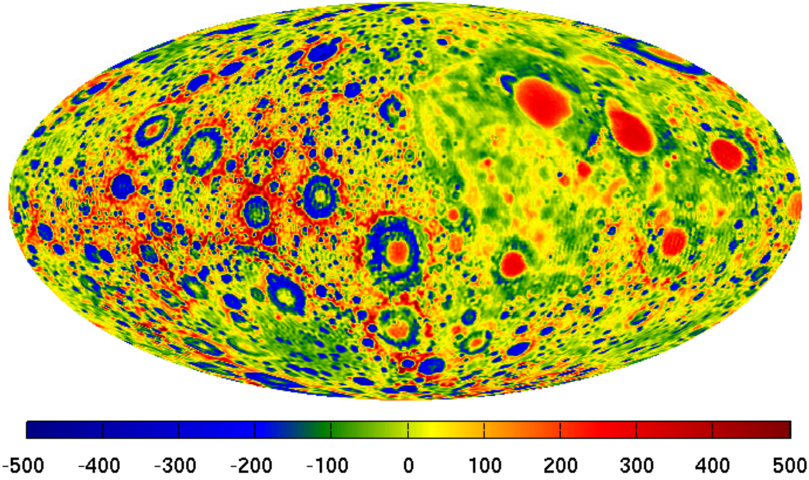
- Development of new capabilities for the Bernese GNSS Software
  - generalization of the force model
  - 1-way and 2-way Doppler modeling
- GRAIL orbit determination results based on Doppler and KBRR data comparable with other groups (if using "good" background model)
- Possible application to other planetary missions.
- Orbit parametrization, arc-length and data screening need to be optimized for robustness.
- Several iterations over gravity field might be necessary for a fully independent solution.
- Pseudo-stochastic orbit parametrization allows for "Bernese" lunar gravity fields without sophisticated background models (SRP is still a limiting factor).



AIUB-GRL200A: Free-air gravity anomalies [mGal]

Thank you!